PDAE
From the field, for the field.

FOCUSING ON MATHEMATICAL REASONING: TRANSITIONING TO THE 2014 GED® TEST

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Session Objectives

- Identify content of the Mathematics module of the 2014 GED® test

- Explore essential mathematical practices and behaviors

- Discuss beginning strategies for the classroom

- Identify resources that support the transition to the next generation assessment

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The 2014 GED® test will . . .

DREAM MORE

- Align with *college and work expectations*
  - Rigorous content and application of knowledge
  - Performance standards supported by validity evidence
- Provide *evidence of readiness*
  - An **indicator of readiness** to enter workforce training programs or postsecondary education
  - Actionable information about a candidate's academic **strengths and weaknesses**
- Provide information about a candidate's *strengths and areas of developmental need*

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The 2014 GED® assessment: Key facts

Attribute	2002 Test	2014 Test
Launch date	January 2002	January 2, 2014
Languages at launch	English	English and Spanish
Delivery mode	Paper and Computer	Computer (Paper for certain limited accommodations)
Delivery locations	Secure testing centers	Secure testing centers
Content	High school content determined by survey of content committees	High school content aligned to career and college-readiness expectations
Passing standard	Set through national standardization and norming study	Set through national standardization and norming study with participation of panels of key stakeholders

The new GED® assessment: Key facts

Attribute	2002 Test	2014 Test
Performance feedback	Scaled score and percentile rank	Scaled score and percentile rank plus information on skills, competencies, and study direction for both high school equivalency and career- and college-readiness
Annual testing opportunities	3 per year (more in last year of 2002 Series)	3 per year (more beginning in 2015 as additional test forms become available)

What will the new GED® test look like?

| Module	2002 Test Testing Time	2014 Test Testing Time
Reasoning Through Language Arts	185 minutes	150 minutes in 3 sections: [25 min + 45 min ER] + [10 min. break] + [70 min] ~51 items – 65 raw score points
Mathematics	90 minutes	90 minutes in 2 sections: [~12 min non-calc] + [~78 min] ~46 items – 49 raw score points
Science	80 minutes	90 minutes in 1 section ~34 items – 40 raw score points
Social Studies	70 minutes	90 minutes in 2 sections: [65 min + 25 min ER] ~35 items – 44 raw score points
Total Battery	~ 7 hours	~ 7 hours

New Score Scale

- Test scaled scores will range from approximately 100 to approximately 200 on each of the four content modules
- The Passing Standard for high school equivalency will be set at a scaled score of 150 for each module
- A minimum score of 600 (150 on each of the four modules) will be required to meet the battery Passing Standard
- The test will be standardized and normed on a nationally representative sample of graduating high school seniors in summer 2013

Tools for Educators

| Assessment Guide for Educators | 2014 GED® Test Item Samplers |
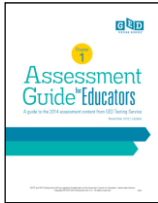
Download the guides at www.GEDtestingservice.com/assessment
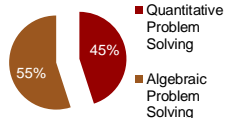
Mathematical Reasoning
What's New?

2002 Series GED® test	2014 GED® test

• One test with two parts, one of which allowed use of calculator
• Content
  • 25-30% Number Operations, Number Sense
  • 25-30% Measurement and Geometry
  • 25-30% Data, Statistics, and Probability
  • 25-30% Algebra, Functions and Patterns
• Casio fx260-Solar
	• One test with calculator allowed on most items
	• Content
	  • 45% - Quantitative Problem Solving
	    • Number operations
	    • Geometric thinking
	  • 55% - Algebraic Problem Solving
	• Texas Instruments - TI 30XS
	• Integration of mathematical practices

Mathematical Reasoning

• Some items require
  • procedural skill
  • fluency
  • problem solving
• Presented in academic and workforce contexts
• Statistics and data interpretation standards are also included in other tests

■ Quantitative Problem Solving
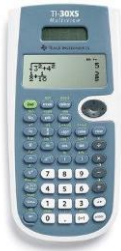■ Algebraic Problem Solving

55%   45%

What's new on the 2014 Mathematical Reasoning Test?

• Identify absolute value of a rational number
• Determine when a numerical expression is undefined
• Factor polynomial expressions
• Solve linear inequalities
• Identify or graph the solution to a one variable linear inequality
• Solve real-world problems involving inequalities
• Write linear inequalities to represent context
• Represent or identify a function in a table or graph

What's **not** on the 2014 Mathematical Reasoning Test?

- Select the appropriate operations to solve problems
- Relate basic arithmetic operations to one another
- Use estimation to solve problems and assess the reasonableness of an answer
- Identify and select appropriate units of metric and customary measures
- Read and interpret scales, meters, and gauges
- Compare and contrast different sets of data on the basis of measures of central tendency
- Recognize and use direct and indirect variation

It's Your Turn!

Let's Have Some Fun!

http://education.ti.com/en/us/products/calculators/scientific-calculators/ti-30xs-multiview/classroom-activities/activities-exchange

Mathematical Practices

- Practices
  - Building solution pathways and lines of reasoning
  - Abstracting problems
  - Furthering lines of reasoning
  - Mathematical fluency
  - Evaluating reasoning and solution pathways

- Most practices are not specific to any one particular area of mathematics content

2002

Ms. Nguyen is a real estate agent. One of her clients is considering buying a house in the Silver Lakes area, where 6 houses have recently sold for the following amounts: $160,000; $150,000; $185,000; $180,000; $145,000; $190,000. What should Ms. Nguyen report as the **Median** price of these houses?

1) $160,000
2) $170,000
3) $180,000
4) $190,000
5) Not enough information is given.

Note: Method for determining median was provided in the test booklet.

2014

Solution Pathways = Problem Solving

Polya's Four Steps to Problem Solving

- Understand the problem
- Devise a plan
- Carry out the plan
- Look back (reflect)
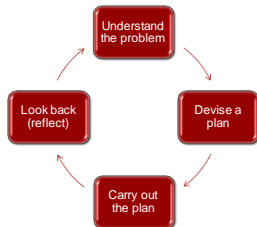
Polya, George. *How To Solve It,* 2nd ed. (1957). Princeton University Press.

Value of Teaching with Problems

- Places students' attention on mathematical ideas
- Develops "mathematical power"
- Develops students' beliefs that they are capable of doing mathematics and that it makes sense
- Provides ongoing assessment data that can be used to make instructional decisions
- Allows an entry point for a wide range of students

SOLVE a Problem

S tudy the problem (What am I trying to find?)

O rganize the facts (What do I know?)

L ine up a plan (What steps will I take?)

V erify your plan with action (How will I carry out my plan?)

E xamine the results (Does my answer make sense? If not, rework.)
  ✓ Always double check!

S = <u>Study</u> the problem

What is the problem asking me to do?
Find the question.

We are going to practice SOLVE with this one!

Each week, Bob gets paid $20 per hour for his first 40 hours of work, plus $30 per hour for every hour worked over 40 hours. Last month, Bob made an additional $240 in overtime wages. <u>If Bob works 55 hours this week, how much will he earn?</u>

O = <u>Organize</u> the Facts

What facts are provided in order for you to solve the problem?

- Identify each fact.
- Eliminate unnecessary facts.
- List all necessary facts.

Each week, Bob gets paid <u>$20 per hour</u> for his <u>first 40 hours</u> of work, plus <u>$30 per hour</u> for <u>every hour</u> worked <u>over 40</u> hours. Last month, Bob made an ~~additional $240~~ in overtime wages. If Bob works <u>55 hours</u> this week, how much will he earn?

L = Line Up a Plan

- Select the operations to use.
- State the plan/strategy that you will use in words.

I will use a multi-step approach. First, I will multiply the number of regular work hours by the regular hourly rate. Next, I will multiply the number of hours of overtime by the overtime rate. To obtain Bob's total weekly salary, I will add the total amount earned for his regular salary plus his overtime salary.

A Few Problem-Solving Strategies

- Look for patterns
- Consider all possibilities
- Make an organized list
- Draw a picture
- Guess and check
- Write an equation
- Construct a table or graph
- Act it out
- Use objects
- Work backward
- Solve a simpler (or similar) problem

V = <u>Verify</u> Your Plan

$20.00
x 40
$800.00

Regular Wages

$ 800.00
+ 450.00
$1250.00

Regular Salary

Overtime Salary

$30.00
x 15
$450.00

Hours Overtime

Total Overtime Salary

Total Weekly Salary

E = <u>Examine</u> the Results
(Is it reasonable? Does it make sense? Is it accurate?)

$1250.00 **IS** reasonable because it is more than Bob's average weekly salary. Also, the answer is a whole number because all of the facts were whole numbers ending in zeros. Therefore, Bob made $1250.00 in salary for the week.

Quantitative Reasoning

• Represents 45% of the items on the test
  • Number operations/number sense
  • Geometric thinking
  • Statistics and data representation (also included on GED® Social Studies test and GED® Science test)

• Some items test procedural skill and fluency as well as problem solving

• Items are presented in academic and workforce contexts

Quantitative Problem Solving Assessment Targets

- Order and compute with rational numbers; simplify numerical expressions (number sense and number operations)

"Calculators can only calculate - they cannot do mathematics."
-- John A. Van de Walle

- Quantitative reasoning to solve problems involving rational numbers (one-step and multi-step word problems)

- Solving problems involving a proportional relationship (rate, ratio, and percent problems)
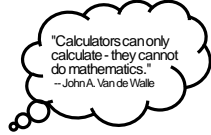
Focus on Geometry

Students need proficiency in basic measurement and geometric thinking skills:
- Use concepts
- Use spatial visualization
- Select appropriate units of measure
- Identify and define different types of geometric figures
- Predict impact of change on perimeter, area and volume of figures
- Compute surface area and volume of composite 3-D geometric figures, given formulas as needed

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The Van Hiele Theory

- Level 1: Visualization
- Level 2: Analyze
- Level 3: Informal Deduction
- Level 4: Formal Deduction
- Level 5: Rigor

Visualization
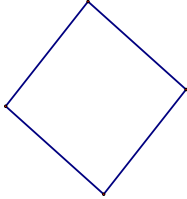
- Recognize and name shapes by appearance
- Do not recognize properties or if they do, do not use them for sorting or recognition
- May not recognize shape in different orientation (e.g., shape at right not recognized as square)

Implications for Instruction - Visualization

- Provide activities that have students sort shapes, identify and describe shapes (e.g., Venn diagrams)
- Have students use manipulatives
- Build and draw shapes
- Put together and take apart shapes
- Make sure students see shapes in different orientations
- Make sure students see different sizes of each shape
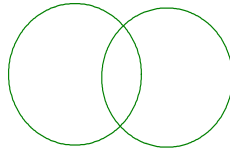
Analysis
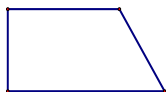
- Can identify some properties of shapes
- Use appropriate vocabulary
- Cannot explain relationship between shape and properties (e.g., why is second shape not a rectangle?)

Implications for Instruction - Analysis
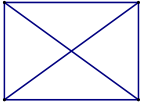
- Work with manipulatives
- Define properties, make measurements, and look for patterns
- Explore what happens if a measurement or property is changed
- Discuss what defines a shape
- Use activities emphasize classes of shapes and their properties
- Classify shapes based on lists of properties

Informal Deduction
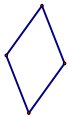
- Can see relationships of properties within shapes
- Recognize interrelationships among shapes or classes of shapes (e.g., where does a rhombus fit among all quadrilaterals?)
- Can follow informal proofs (e.g., every square is a rhombus because all sides are congruent)

Deduction

- Usually not reached before high school; maybe not until college

- Can construct proofs

- Understand the importance of deduction

- Understand how postulates, axioms, and definitions are used in proofs
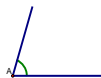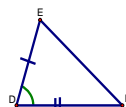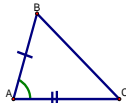
Geometric Reasoning

- Seeking Relationships
- Checking Effects of Transformations
- Generalizing Geometric Ideas
  - Conjecturing about the "always" & "every"
  - Testing the conjecture
  - Drawing a conclusion about the conjecture
  - Making a convincing argument
- Balancing Exploration with Deduction
  - Exploring structured by one or more explicit limitation/restriction
 - Taking stock of what is being learned through the exploration

What Are Some Ways to Solve These Problems?

- Each section of the Bradley Center holds 258 people. There are 72 sections. If all of the seats are filled for a Buck's game, how many people would there be?

- How many bows can you make from 3 2/3 meters of ribbon if you need 2/5 meters of ribbon to make each bow?

Would the Same Methods Work?

- Each section of the Bradley Center holds p people. There are s sections. If all of the seats are filled for a Buck's game, how many people would there be?

- How many bows can you make from r meters of ribbon if you need m meters of ribbon to make each bow?
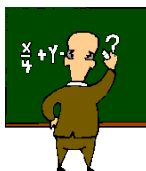
Algebraic Thinking in Adult Education

- Create opportunities for algebraic thinking as a part of regular instruction

- Integrate elements of algebraic thinking into arithmetic instruction
  - Acquiring symbolic language
  - Recognizing patterns and making generalizations

- Reorganize formal algebra instruction to emphasize its applications

Adapted from National Institute for Literacy, *Algebraic Thinking in Adult Education*, Washington, DC 20006
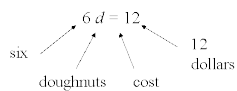
Some Big Ideas in Algebra

- Variable
- Symbolic Notation
- Equality
- Ratio and Proportion
- Pattern Generalization
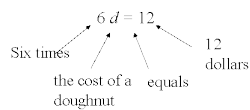- Equations and Inequalities
- Multiple Representations of Functions

Variable

Some students believe that letters represent particular objects or abbreviated words

Wrong Interpretation: $6\,d = 12$ — six, doughnuts, cost, 12 dollars

Correct Interpretation: $6\,d = 12$ — Six times, the cost of a doughnut, equals, 12 dollars

Symbolic Notation

A Few Examples

Sign	Arithmetic	Algebra
= (equal)	. . . And the answer is	Equivalence between two quantities
+	Addition operation	Positive number
-	Subtraction operation	Negative number

Confusion About the Equal Sign

The equal sign stands for balance or equality.
The concept of balance can be used to reinforce the idea of equality – both sides of the number sentence need to be the same, the equation needs to balance.
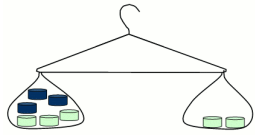
Patterns – Thinking Algebraically

• Finding patterns

• Describing patterns

• Explaining patterns

• Predicting with patterns

Algebra RULES!

Teaching Patterns

Banquet Tables

Arrangement 1   Arrangement 2   Arrangement 3

- Arrangement 1 seats four people. How many people can be seated at Arrangement 100?

Use Multiple Representations

- Represent problems using symbols, expressions, and equations, tables, and graphs
- Model real-world situations
- Complete problems different ways (flexibility in problem solving)

Effective Questions

Ask challenging, well-crafted, ended questions, such as:

- What would happen if . . . ?
- What would have to happen for . . . ?
- What happens when . . . ?
- How could you . . . ?
- Can you explain why you decided . . . ?

Teacher Responses

- Phrases to Use
  - I'm not sure I understand, could you show me an example of ... ?
  - What do you think the next step should be?
  - Where would you use ... ?
  - Could _____ be an answer?
  - How do you know you are correct?

- Phrases to Avoid
  - Let me show you how to do this.
  - That's not correct.
  - I'm not sure you want to do that.

Algebra RULES!

Associated Press Poll

- People have a "love-hate" relationship with mathematics
  - Twice as many people hated it as any other school subject
  - It was also voted the most popular subject

Math journals help students to . . .

- Be aware of what they do and do not know
- Make use of prior knowledge
- Identify their mathematical questions
- Develop their ability to problem solve
- Monitor their own progress
- Make connections
- Communicate more precisely

Make It Real!

**Mathematics is like a video game;
If you just sit and watch,
You're wasting your time.**

Algebraic thinking . . .

Involves the connection between all learning levels.

- Concrete
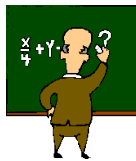
- Representational (semi-concrete)

- Abstract $a^2 + b^2 = c^2$

Strategies for Teaching Algebra

- Modeling function approach – teaching big ideas
- Employing effective questioning and responses
- Using manipulatives

5 Basics of Number Knowledge Essential for Algebra Learning

• Understanding equality
• Recognizing the operations
• Using a wide range of numbers
• Understanding important properties of numbers
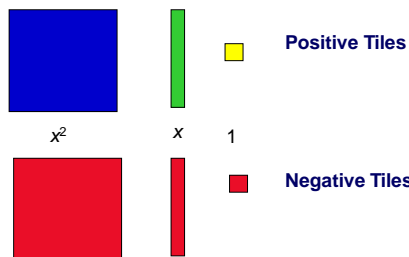• Describing patterns and functions

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Manipulatives for Algebra "C" of CRA)

• Students with access to virtual manipulatives achieved higher gains than those students taught without manipulatives.

• Students using hands-on and manipulatives were able to explain the how and why of algebraic problem solving.

algebra tiles, algetiles, math tiles, virtual tiles

Introduction to Algebra Tiles

Positive Tiles

x^2 x 1

Negative Tiles

Remember, they could be called x, y, b, t, etc.

Big Ideas Using Algebra Tiles

- Add and subtract integers
- Model linear expressions
- Solve linear equations
- Simplify polynomials
- Solve equations for unknown variable
- Multiply and divide polynomials
- Complete the square
- Investigate

Use Algebra Tiles to Model Addition of Integers

- Addition is "combining."
- Combining involves the forming and removing of zero pairs.
- Remember, an integer is a number with no fractional part.

Use Algebra Tiles to Model Integer Subtraction

- Subtraction can be interpreted as "take-away."

- Subtraction can also be thought of as "adding the opposite."

Use Algebra Tiles to Combine Polynomials

- "Simplify" means to combine like terms and complete all operations.

Terms in an expression are **like terms** if they have identical variable parts.
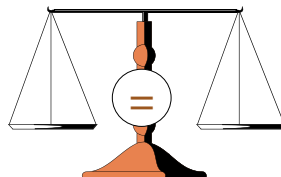
You can **combine terms** that are alike.

You **cannot combine** terms that are unalike.

More Polynomials – Set It Up and Solve!

- $(2x^2 + 5x - 3) + (-x^2 + 2x + 5)$

- $(2x^2 - 2x + 3) - (3x^2 + 3x - 2)$

Solve Linear Equations

- Equation solving model is based on two ideas
  - Equations are unchanged if equivalent amounts are added to each side of the equation.
  - Variables can be isolated by using zero pairs.

Let's Collect Tiles!

The Rules!
- Big squares can't touch little squares.
- Little squares should all be together.
- Tiles should always be in a rectangular array.

$2 x^2 + 7x + 6$
Which looks best?
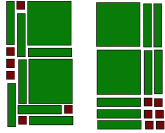
Algebra Tiles – Time to Collect Tiles!

- $x^2 + 6x + 8$

- $x^2 - 4x + 3$

- $x^2 + 7x + 6$

- $2x^2 + 7x + 6$

Multiplying Polynomials

It's just like figuring area!
- Place one term at the top of the grid
- Place the second term on the side of the grid
- Maintain straight lines when filling in the grid
- The inner grid is your answer!

Multiplying Polynomials

- $(x + 2) (x + 1) =$

- $(x + 5) (x + 3) =$

- $(2x + 2) (2x + 1) =$

Binomials are FUN!

Your Turn!

Dividing Polynomials

Think about it! It's just like multiplication – only with different known items
- Place the divisor on the side of the grid
- Create the inner grid as a rectangular array (the dividend)
- Identify the missing term (quotient) that completes the top of the grid
- You have your answer!

Dividing Polynomials

$$\frac{x^2 + 7x + 6}{x + 1}$$

$$\frac{2x^2 + 5x - 3}{x + 3}$$

$$\frac{x^2 - x - 2}{x - 2}$$

$$\frac{x^2 + x - 6}{x + 3}$$

Factoring Polynomials

- Algebra tiles can be used to factor polynomials. Use tiles and the frame to represent the problem.
- Use the tiles to fill in the array so as to form a rectangle inside the frame.
- Be prepared to use zero pairs (when needed) to fill in the array.
- Solve!

Factoring Polynomials

$x^2 + x - 6$
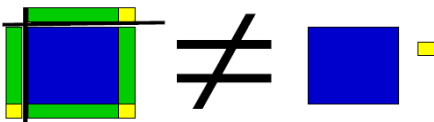
$x^2 - 1$

$x^2 - 4$

$2x^2 - 3x - 2$

$2x^2 + 3x - 3$

$-2x^2 + x + 6$

Algebra RULES!

There's More!

Investigate

- Use algebra tiles to prove that $(x + 1)^2$ and $(x^2 + 1)$ are not equivalent

$(x + 1)^2 = (x+1)(x+1)$
$= x^2 + 2x + 1$

$x^2 + 1$

There's More!

Use Algebra Tiles to Complete the Square
- What is needed to create a perfect square trinomial for $x^2 + 4x + ?$
- Use algebra tiles to create a square. What tiles will be needed to complete the square?
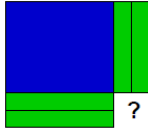
It all leads to connecting mathematical concepts with effective mathematical practices/problem solving

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Real-World Algebra

My Ford Bronco was fitted at the factory with 30 inch diameter tires. That means its speedometer is calibrated for 30 inch diameter tires. I "enhanced" the vehicle with All Terrain tires that have a 31 inch diameter. How will this change the speedometer readings? Specifically, assuming the speedometer was accurate in the first place, what should I make the speedometer read as I drive with my 31 inch tires so that the actual speed is 55 mph?

CTL Resources for Algebra. The Department of Mathematics. Education University of Georgia
http://jwilson.coe.uga.edu/ctl/resources/Algebra/Algebra.html

Real-World Math

The Futures Channel
http://www.thefutureschannel.com/algebra/algebra_real_world_movies.php

Real-World Math
http://www.realworldmath.org/

Get the Math
http://www.thirteen.org/get-the-math/

Math in the News
http://www.media4math.com/MathInTheNews.asp

One Last Item!

As a group, determine the top three things you WILL do to build a meaningful mathematical reasoning program for ALL students as you transition to the 2014 GED Mathematics Test!!

SUCCESS

PDAE
From the field, for the field.

Thank you for being with us today!

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The IPDAE project is supported with funds provided through the Adult and Family Literacy Act, Division of Career and Adult Education, Florida Department of Education.